1 **a** $(2,-4) \to (2+(-4), 2-(-4)) = (-2,6)$

b $(2,-4) \to (2(2)+3(-4), 3(2)-4(-4)) = (-8,22)$

c $(2,-4) \to (3(2)-5(-4), 2) = (26,2)$

d $(2,-4) \to (-4,-2)$

2 **a** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0\times 2+1\times 3 \\ 1\times 2+0\times 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Therefore $(2,3) \to (3,2)$.

b $\begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2\times 2+0\times 3 \\ 0\times 2+3\times 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 9 \end{bmatrix}$

Therefore $(2,3) \to (-4,9)$.

c $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times 2+2\times 3 \\ 0\times 2+1\times 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$

Therefore $(2,3) \to (8,3)$.

d $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2\times 2+1\times 3 \\ 1\times 2+3\times 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

Therefore $(2,3) \to (7,11)$.

3 **a** The linear transformation can be written as

$$x' = 2x + 3y$$
$$y' = 4x + 5y$$

so the transformation matrix is

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}.$$

b The linear transformation can be written as

$$x' = 11x - 3y$$
$$y' = 3x - 8y$$

so the transformation matrix is

$$\begin{bmatrix} 11 & -3 \\ 3 & -8 \end{bmatrix}.$$

c The linear transformation can be written as

$$x' = 2x + 0y$$
$$y' = x - 3y$$

so the transformation matrix is

$$\begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix}.$$

d The linear transformation can be written as

$$x' = 0x + 1y$$
$$y' = -1x + 0y$$

so the transformation matrix is

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

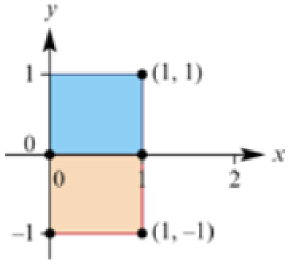
4 For each of these questions we multiply the transformation matrix by a rectangular matrix whose columns are the coordinates of each vertex.

a $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & -1 \end{bmatrix}$

The columns then give the required points:

$(0, 0), (0, -1), (1, 0), (1, -1)$.

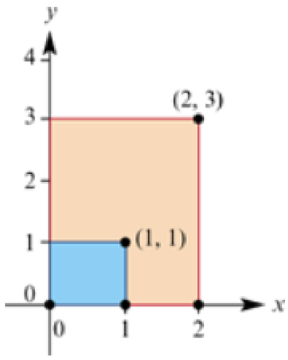
The square is shown in blue, and its image in red.



b $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 2 \\ 0 & 0 & 3 & 3 \end{bmatrix}$

The columns then give the required points:

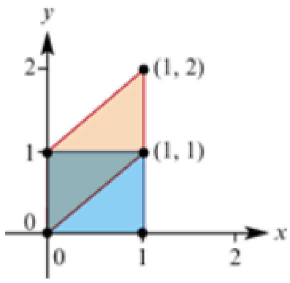
$(0, 0), (2, 0), (0, 3), (2, 3)$. The square is shown in blue, and its image in red.



c $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 0 & 0 & 3 & 3 \end{bmatrix}$

The columns then give the required points:

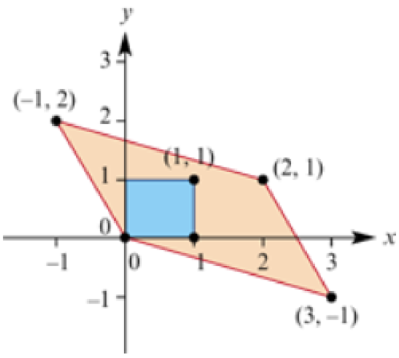
$(0, 0), (1, 1), (0, 1), (1, 2)$. The square is shown in blue, and its image in red.



d $\begin{bmatrix} -1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 3 & 2 \\ 0 & 2 & -1 & 1 \end{bmatrix}$

The columns then give the required points:

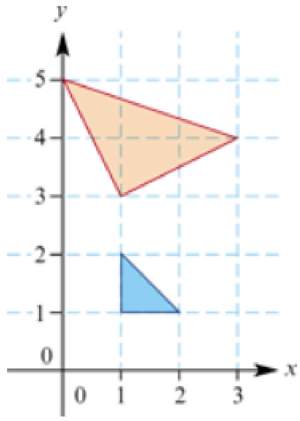
$(0, 0), (-1, 2), (3, -1), (2, 1)$. The original triangle is shown in blue, and its image in red.



5 We multiply the transformation matrix by a rectangular matrix whose columns are the coordinates of each vertex. This gives, $\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 3 & 5 & 4 \end{bmatrix}$

The columns then give the required points:

$(1,3), (0,5), (3,4)$. The square is shown in blue, and its image in red.



6 The image of $(1,0)$ is $(3,4)$. The image of $(0,1)$ is $(5,6)$. Write these images as the column of a matrix, $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$.

Therefore $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \end{bmatrix}$

so that $(-2,4) \to (14,16)$.

7 The image of $(1,0)$ is $(-3,2)$. The image of $(0,1)$ is $(1,-1)$. Write these images as the column of a matrix, $\begin{bmatrix} -3 & 1 \\ 2 & -1 \end{bmatrix}$. Therefore $\begin{bmatrix} -3 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

so that $(2,3) \to (-3,1)$.

8 **a** $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$.

b $\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}$ or $\begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}$.

c $\begin{bmatrix} 1 & -2 \\ -1 & -3 \end{bmatrix}$ or $\begin{bmatrix} -2 & 1 \\ -3 & -1 \end{bmatrix}$.